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Spalding

32

A PRACTICAL TREATISE
On the Adjustment of the
THEODOLITE,
AND
A FEW HINTS TO YOUNG SURVEYORS.

BY ROBERT SPALDING,

Serjeant, Royal Sappers and Miners.

85

Second Edition.

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BIBLIOTHECA BODLEIANA

283.

PREFACE.

In compiling the following pages, my chief aim has been to explain as clearly as possible, the use and adjustment of the Theodolite, and to remove a few of the many difficulties which occur to the young surveyor. Those who expect an elaborate treatise may meet with some disappointment, since the entire body of the work is but a few notes and ideas hastily written, and as hastily combined into the form they at present assume.

On the trigonometrical survey of Ireland I had numberless opportunities of witnessing the difficulties which are met with in the survey of the face of a rugged and mountainous country, I trust then that seven years experience will be a sufficient recommendation and apology for the introduction of this pamphlet to the notice of my military brethren of the Royal Sappers and Miners, who may be destined for the survey of Ireland.

Wishing to be as concise as possible, I have studiously withheld every observation which could not be said to appertain decidedly to Land Surveying.

Surveying forming a principal branch of the studies pursued by the Honorable East India Company's Sappers and Miners, I also feel great pleasure in presenting to their notice this little work, so well calculated to remove the difficulties which young surveyors will necessarily have to encounter.

Disclaiming all pecuniary considerations, and wishing that this humble attempt may be found as useful as I design it to be,

I subscribe myself,

The well-wisher of all branches of the Service,

ROBERT SPALDING,

Serjeant, Royal Sappers and Miners.

Chatham Barracks,
31st March, 1832.

TREATISE

ON THE

ADJUSTMENT of the THEODOLITE.

INSTRUCTIONS FOR ADJUSTING AND USING A SEVEN OR EIGHT INCH THEODOLITE.

Previous to commencing operations I would strongly recommend the student to make himself acquainted with reading the degrees, minutes, &c. on the instrument, a knowledge of which will easily be obtained from a master of the instrument with a little application.

The Theodolite should undergo the following adjustments.

1st. The levels attached to the horizontal plate. They should be placed perpendicularly across a pair of foot screws, in which position they will be parallel with the other two screws. Level the instrument, and when the upper plate is turned half round, should the levels not then stand as before, but shew one end higher than the other, half the difference thus shewn is to be corrected by the levelling screws, and the other half by the capstan screws, at either end of the level, when one of these screws is slackened the other must be driven close, that the level may have no play. Several reversions will be necessary to ensure accuracy, without which, especially in taking angles on remote objects the instrument cannot be depended upon.

2nd. The Level attached to the Telescope.

The first thing necessary in the adjustment of this level is to place it in the plane of the telescope; to effect this, clamp the horizontal plates of the instrument, and also the vertical arch, and by the motion of the tangent screw, level the telescope; when steady in this position, turn the telescope carefully in the

Ys[*], several degrees right and left of the vertical plane in which it now stands, and should the bubble of the level remain in the centre of the tube during these pendulous motions, the level is already in the plane of the telescope, but should it run to either end an adjustment is necessary, which is effected thus—should the level on being moved to the right shew the end, having the screw for the lateral adjustment too high, that end of the level must be moved to the left, but if the other proves the higher end, the level requires to be moved to the right; several trials will of course be necessary in this as well as all other adjustments of the instrument.

The level must now be placed parallel to the optical axis of the telescope—that is, a line passing through the centre of the eye and object glasses.

The instrument remaining clamped, and the telescope levelled as before, reverse it carefully in the Ys, and in doing this, it will be sufficiently accurate to place the level over the brass connecting the Ys, as the lateral adjustment has been made; should the bubble remain in the center of the tube when reversed, no adjustment is necessary, this however, can only be ascertained by three or four careful reversions, as in the act of reversing there is a great tendency to move the vertical arch; but should one end of the level prove higher than the other, half the error is corrected by the tangent screw of the vertical arch, and the other half by the capstan screw of the level.

3rd. To adjust the line of collimation,[†] place the intersection

---

[*] The Ys are the pieces in which the Telescope rests.

[†] The wires or cross hairs should be as the annexed Fig. L., as it can be easily done, and will answer much better, as you can see that if they are placed as the Fig. D. the line A D will cover the whole length of a station pole, (if very distant) and you never can place the centre of them so well on the object as when fixed as Figure L.

Fig. L.

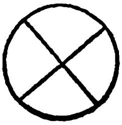
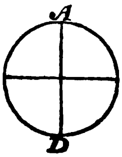


Fig. D.



of the hairs upon a well-defined object, about a mile distant, and turn the telescope carefully in the Ys.

Should the intersection cover the object during the entire revolution, the instrument is already collimated, but if not, an adjustment is necessary; for this purpose, place the intersection of the hairs on a point (such as A in the annexed figure,) in or near some vertical line, turn the telescope quarter round, and if the intersection appears above, and to the left of the point, as at B, Fig. 2, it must be drawn apparently farther in these directions (as the hairs appear in an inverted position) by driving the screws C and D, having previously slackened the opposite screws E and F. This process must be continued by placing each hair successively in a vertical position, until the intersection of the hairs cover the distant point during an entire revolution of the telescope.

Fig 1

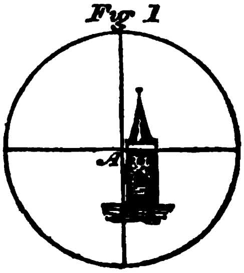
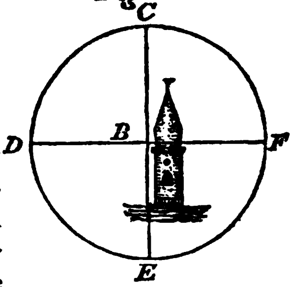
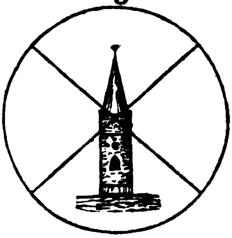


Fig 2



The level being placed over the brass connecting the Ys, bring the hairs in contact with the angles of a square building; should the hairs come in contact with each angle at the same time, the cylinder is in the proper position; if not, it must be moved round, (bearing in mind the effect of inversion) until the hairs are brought in contact with the angles of the building as shown in the annexed figure.

Fig 3



The instrument being adjusted and levelled, adjust the focus of the telescope on an object four or five miles distant, and when the observations are once commenced the focus should not be altered, nor the telescope moved in the Ys, as the arc would be rendered useless thereby. This has been proved by actual observations made in frequent trials; for if you place the intersection of the hairs on a well-defined object, and alter the focus of the telescope, you will perceive that the intersection will differ something from the first observation made.

These directions being attended to, commence on a well-defined object, as in the annexed figure A, about three or four miles distant. then observe three other objects as B, C, and D, dividing the circle as nearly as the objects will admit, proceed to observe all the minor stations to the distance of the second series of triangles, and the primary stations to the distance of 10 or 12 miles, also observing the objects A. B. and C. as they occur in the arc, and closing it on the object D. These four objects will then appear twice on each arc.

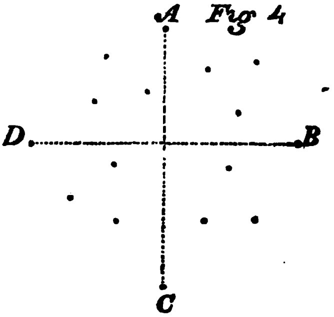


Fig 4

This is to be attended to without any reference to the portion of country to be surveyed by the division to which the surveyor belongs; degrees, minutes, and seconds to be read from each vernier, beginning from the left, and the reading of both to appear in the book.

The arc should be changed 80 or 90 degrees, but not placed to the same number of minutes and seconds which any former arc read.

The instrument to be levelled previous to commencing each arc. Three arcs will be sufficient for the minor objects, and four for the principal ones, in conjunction with which the referring objects A, B, C, and D, are to be observed.

All well-defined permanent objects, such as Church Steeples, Towers, &c. are to be observed. Figure 3 shews the manner in which objects of this nature are intersected when viewed direct.

but when obliquely the centre of the mass which presents itself to view must be intersected.

The vertical arcs are not to be taken at the same time with the horizontal arcs, as by this method it is impossible to avoid errors. Previous to commencing these arcs, place the intersection of the wires upon some perpendicular object, and turn the telescope in the Ys. until the perpendicular object bisects the angle formed by the hairs, after which, it must not be moved (nor the focus altered) during the progress of the present round of vertical arcs. On finishing the first round of vertical arcs, the collimation is to be examined, and if found accurate, the words, " Instrument collimated," to be written on some part of the page on which the vertical arcs are entered. If not found to be accurate, six or eight well-defined objects already observed, must be taken with the telescope inverted, and the words " Telescope inverted," to be written over the observations thus taken.

In taking vertical angles with the telescope inverted, turn the telescope half round in the Ys; to ensure the accuracy of which operation, the intersection of the hairs must be placed upon the object, and the telescope moved until the object is made to bisect the angles of the hairs, as was done previous to commencing the vertical arcs, with the telescope direct.

The angle is then read and booked, and the telescope carefully brought to its direct position, and levelled in order to ascertain the index error.

In order to guard against an error in reading, a second round of vertical arcs will be taken, and that the accuracy of the adjustment of the level attached to the telescope may likewise be tested, the telescope will be reversed: having therefore reversed the telescope in the Ys, and re-levelled the instrument, a second round of vertical arcs will be observed, the telescope examined with respect to collimation, and six or eight objects observed with the telescope in an inverted position, as before directed. The foregoing instructions are likewise applicable to the five-inch or to any Theodolites constructed on similar principles.

INSTRUCTIONS FOR FITTING IN A NEW SET OF WIRES OR CROSS HAIRS.

It often occurs that the wires or cross hairs get broken, by fitting in the eye tube of the telescope, screwing in the brass end that protects the hairs, and in short, there are many ways in which they may be broken; they frequently give way after a severe day's rain, in consequence of being fastened in with gum, but to remedy that, I would recommend that they should be fixed with thick copal varnish; it should always be at hand, together with a piece of silk thread; having these, you may put a new set of wires in and adjust the line of collimation in about a quarter of an hour; the atmosphere will not touch the varnish, and a single hair of the silk will make the wires so as their thickness will not exceed 1000 part of a fine thread of silk; before you put new ones in, the substance that held the old ones should be taken off the cross lines on the ring. Care should be taken not to injure these marks; put a small portion of varnish on the lines marked on the brass, then take about 1½ inch of floss silk, and lay it on, and with a finger of each hand press the varnish and hairs into the marks, at the same time drawing the fingers towards the outside of the ring to straighten them in their places, and put the ring into its position; it is at this period you can please yourself as to the position of the cross hairs, either as figure L or D, p. 6. In the ring there are 8 holes equi-distant from each other, therefore it will admit of being changed either to the vertical or oblique position; proceed then to adjust the line of collimation, as directed in page 7.

Every person who uses a Theodolite should be able to adjust it, and put in new wires, as much valuable time may be lost by his not being able so to do, as he might at the time of an accident occurring, be very distant from a person who could do it. Surveyors frequently carry the instrument when fixed on its stand, sloping over the shoulder; such a practice has a great tendency to bend the centre of the Theodolite, it must be remembered that the spindle is a very delicate part of the instrument; in cleaning them nothing should be used but a piece of fine chamois, with a little olive oil to moisten the corroded parts, (if any) all of which must be wiped clean off again.

Whenever instruments work stiffly, the ball working in the socket should be examined, as it sometimes works out of its place, which causes the plates to act very stiffly; a forked screwdriver is required to screw it up, but before this can be done, all the foot screws should be worked up close to the neck of the socket to loosen the parallel plates. I have seen instruments injured by forcing the upper plate round while the ball was out of its place, this I mention in order that the young surveyor may not fall into the same error.

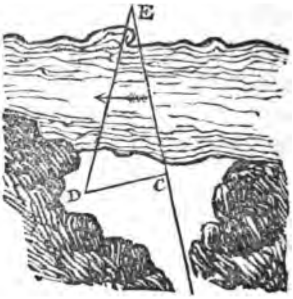
Whenever Theodolites are exposed to wet, they should be dried as soon as possible; the object glass in particular should be well dried, as it is composed of three pieces, and should the damp once penetrate between them, it may keep a person unemployed next day, as it requires a long time to dry.

Although observations cannot be booked, yet lines may be laid out and work prepared in wet weather; in order to keep the rain from the object glass, a small tube, one end to fit on the end of the telescope, the other left with an opening of about a quarter of an inch, (the entire should be about two inches long,) will effectually prevent the rain from drifting against the object glass, as in surveying a mountainous country the surveyor would put up with any inconvenience rather than return without accomplishing his work under any circumstances.

The following will shew the utility of a Theodolite in triangulating ground, interspersed with lakes, rivers, woods, &c. and where it is necessary to measure the principal lines (instead of taking their lengths from paper as is frequently done,) as no other method will give the true area of the several divisions of a country, for it has been fairly proved that a survey done by traverse, with or without the needle, or any other instrument used in surveying, comes far short of the accuracy obtained by triangulation.

In short the traverse surveying may (in almost every case) be dispensed with, as every object on the face of a country may be fixed and laid down with a less accumulation of errors than could be done by a traverse survey. This mode requires so many stations that the errors become numerous, as in no case can an angle be protracted with that degree of accuracy that lines can, but although the traverse survey may be laid aside, still the Theodolite is an instrument indispensably necessary to the surveyor, as it not only furnishes him with horizontal and vertical distances, but also affords him the means of passing any obstacles that he may meet with.

Fig. 5.



Example I. The annexed figure represents a line crossing the river, and it may so happen that it runs in a deep ravine. Here I find it impossible to raise a perpendicular equal to the breadth of the river; if this could be done on an angle of 45°, at the point D, would produce a perpendicular equal to the base, which would give the distance from C to E.

But in the present example as that cannot be accomplished, angles must be used either at D or E that will produce a perpendicular equal to 1¼, 1½, 1¾, 2, 2¼, 2½, 2¾, 3, 3¼, 3½, 3¾, or 4 times the length of the base. The calculation for which is very simple, and may be useful in case the Surveyor may require to increase the proportions.

The Problem is—The base, perpendicular, and contained angle of a right angled triangle given—to find the other angles.

RULE.

Add radius to the logarithm of the less side, and from that sum subtract the logarithm of the greater, the remainder will be the tangent of the angle opposite the less side. Take figure 5 for instance, and the necessary calculations for finding the angles that will produce the side C E, equal to twice the line C D, will be as follows:

Radius + Logarithm of side C D = 12·6532125
C E = 2·9542423

Log. Tan. 9·6989702 = 26° 34′

Angle D E C is 26° 34′, that taken from 90° = 63° 26′ for the angle C D E.

The base may be any number that will suit the ground, but it will not be safe to use an angle that will produce a perpendicular of more than 4 times the base, as a small error in the angle C D E would cause a great difference in the length of the line C D. The base C D in this case is 450, and the line C E 900.

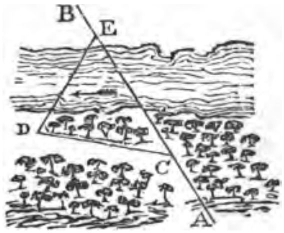
The following Table will answer in any case where a perpendicular can be raised. In the first column of the Table is the length of the base, or line turned off to clear the obstacle; the second column shews the length of the perpendicular, or in other words, the number of times the base is contained in the perpendicular; the remaining columns shew the angles to be used for finding these proportions; that marked E shews the angle to be used at the point E, and that under D the angle to be used at the point D. For Ex.—Say it is required to get a perpendicular 3½ times the length of the base; by the Table I find the angle at E will be 15 deg. 57 min., and at D will be 74 deg. 3 min. if the line C D had been 1, then would the line C E be equal to 3½.

Table.

		E		D	
C D	C E	Deg.	Min.	Deg.	Min.
1	1¼	38	39	51	21
	1½	33	41	56	19
	1¾	29	45	60	15
	2	26	34	63	26
	2¼	23	58	66	2
	2½	21	48	68	12
	2¾	19	59	70	1
	3	18	26	71	34
	3¼	17	6	72	54
	3½	15	57	74	3
	3¾	14	56	75	4
	4	14	2	75	58

When it is necessary to measure a line it will be much better to complete it from one end by clearing obstacles, and carrying on until completed; it is sometimes done by chaining from both ends when any obstruction occurs on it, leaving that portion which cannot be measured by the chain to be determined by the person laying down the work, which is not only uncertain, but very troublesome in registering the contents, and plotting from both ends of a line. It will often occur in surveying, that you cannot turn off at right angles from your main line, on arriving at a Lake, River, Wood, Town, or Village, in consequence of obstructions lying right and left of the line, the annexed Figure will explain the manner of removing this inconvenience.

Fig. 6.



Example II. On arriving at the point C, it was impossible to turn off at right angles, on account of woods, in this case, use any angle for turning off, that will admit of your chaining its direction a distance equal to the breadth of the obstacle. Suppose the angleE C D measures 63° 20′, consequently the sum of the other two angles will be 116° 40′, that being divided by two, will give the other two angles, which will be each 58° 20, from which it is evident that if two angles are alike, their opposite sides will be alike also.

I have to remark, that in all these methods of turning off, it will be better to use the angle C E D, as your principal line will afford a better base to set the angle off from; in this case, you can place your instrument at the point E, set off the required angle, at the same time your assistants are chaining the line C D, and when they arrive in the direction of your angle, give them the signal to stop; one of them will then place a station staff on the chain, according to your direction, which will, consequently, be in the point D. All these plans have been found to answer well on the Survey of Ireland, so much so, that in some of the triangles for the content of parishes, whose sides were from three to five miles in length, and interspersed with Lakes, Rivers, Rocks, and

Ravines, they were actually measured to within from three to ten links of the calculated length, a distance imperceptible on a scale of six inches to a mile. It is also to be observed, that all these distances were measured by levelling small portions of the chain (in uneven or hilly ground,) and using an iron plummet pointed with steel, as in many places it would have been impossible to have placed an instrument to take an angle of elevation or depression for the purpose of reducing the line to the plane of the horizon.

ON THE ADJUSTMENT OF THE CHAIN, WITH INSTRUCTIONS FOR CORRECTING LINES WHEN IT IS IN ERROR.

The chain used in surveying is 66 feet in length, or 4 poles of 16½ feet, made of good wire, and consists of long links connected together by rings, each link is 7·92 inches, consequently there are 100 of these links in the 66 feet. The rings between the links serve for adjusting the chain, as it very seldom remains two days alike when used; it is numbered from each end towards the centre by pieces of brass marked with notches, corresponding with the number of tens they represent; the centre, or 50, being marked with a round one.

Great pains must be taken to keep the chain in good adjustment. Before you begin a survey, a standard distance of 66 feet should be laid down convenient to your quarters, in order that you can compare your chain every morning, and register its error, if any, in your Field-Book, for all work done by it on the day before; in omitting to note the error of the chain, when chaining principal lines, will perhaps cause a Surveyor to re-survey the same, as on comparing the measured distance with the calculation, the difference may be more than one link in a thousand, and any lines whose difference exceed this must be re-measured. For instance, let a line be chained and it turns out to be 32400 links, (a distance not unfrequent) next morning the chain was long ·1 of a link, then ½ of the same would be taken; in this case you would have to add

16 links to the length of the line which will make it 32416; now this would affect the content of the triangle very much, and with nice instruments plotting on a scale of 6 inches to a mile, would cause the work to twist. When a person tries his chain, the error should be measured by a small scale of brass or iron, with two or three tenths of a link on it, and one divided into ten; by this the true error can be ascertained, and as I before stated ½ of it will be multiplied by the length of the lines measured by the chain having such an error.

Suppose I commence to register from the Field-Book, and find in pages 3 and 4 that the chain was long ·12 of a link; now to enter such work without paying attention to chain errors, the areas so found would differ considerably from the truth. Say in a triangle the sides of which are 9763, 11720, and 12343, the area will be 5357·108 square chains; but with sides corrected, instead of their being as above, they will be 9769, 11727 and 12350.

$$\text{Cor.}\left\{\begin{array}{l} 9763 \times {\cdot}06 = 5{\cdot}8578 \qquad\quad + \; 9763 = 9769 \\ 11720 \times {\cdot}06 = 7{\cdot}0320 \text{ links} + 11720 = 11727 \\ 12343 \times {\cdot}06 = 7{\cdot}4058 \text{ links} + 12343 = 12350 \end{array}\right\}\begin{array}{l}\text{True} \\ \text{Length.}\end{array}$$

and the area of these corrected sides will be 5363·500 sq. chains.

Now here is a difference of 6·39 square chains, which may convince any one how necessary it is to have a proper standard distance laid down, instead of using wooden pegs,* (as recommended by a late Author on Surveying. See Burr's Treatise on Practical Surveying, p. 34.) Let two stones, about half a cubic foot each, be sunk in the ground, and properly marked with a fine cold chissel, in the shape of a +; the centre of which will agree with the standard distance. The distances should be laid down in more than one place, in every parish, lest any one should derange them.

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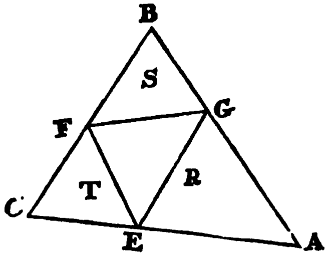
* Wooden pegs would be but a poor shift.

CHAINING AND KEEPING THE BOOK.

Stations should be erected on the most conspicuous ground, and avoid as much as possible making small Triangles, as in laying down a number of short lines the protraction has every chance of being twisted, at the same time it causes many more observations to be taken. When Triangles are from 4 to 5 miles a side, it will be better to use intermediate poles (as in the annexed figure) wherever you find it necessary, and their position in the true line, can be found previous to commencing the chain survey; by using a Theodolite at the extreme stations, or by trials between them.

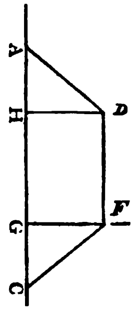
Fig. 7.



The first thing to be done on arriving at a station, is to enter the names of the party, the letter or No. of Theodolite, and No. of chain ; stating whether the instrument is in adjustment or not, and leave a blank opposite the chain, to enter the state it may be found in next day, when applied to the standard distance. Every page of the Field Book, must contain the day of the month; but the first page of a day's work only requires to have the Surveyor's name, party, theodolite, chain, &c.

If the line is a principal one, it will be better to rule two lines in the most convenient part of the page, and set the letter and name of the trigonometrical station at the bottom of the line, and that of the one you are going to, on the top; by having your line so you may fix objects on both sides of it; the line should be well laid out by an instrument, otherwise the interior of the work will not fit well, if any of the points the smaller lines run from should be out of the true line; besides which, the lines will all be too long. When an object is to be fixed on either side of the line, care must be taken that your offsets are perpendicular, and in no case should they exceed two chains; and then, if there are many wanting, the objects must be fixed by some other figure, (such as two perpendiculars raised with a theodolite, and connected by a line taking in the boundary, and the lines C F and D A must be run as in the annexed figure, to the main line, to ensure accuracy, as you may see that the line F D would answer at several points on the line F G, in consequence of its meeting the perpendicular nearly at right angles. When the person dragging the chain has put his last arrow into the ground, he goes on once without one, and then waits till the person directing comes up, who gives him the arrows, one of which is left in the ground as a point to start from, and in no case should the chainman proceed till he counts his arrows, as sometimes the person directing may leave one in the ground. Whenever an error of this sort occurs, the chainmen must go back and re-measure from an object or mark noted on the line, as it will never answer to patch a line by guess, for you cannot be certain which of the chainmen lost or left it in the ground. Sometimes it happens that 75 is read for 25, and 36 for 64, and so on. Whenever a line is at a close, let the chain be drawn past the station, lest the odd links may be read from the reverse end. The surveyor should pay particular attention to the lining of the chain, and he should give the person dragging it a point to line on, as when the person directing loses the forward station it will serve as a check on the chain, having the proper direction. Mistakes often occur in borrowing ten or twenty links, when the chain is so that the arrow would be in the centre of a ditch or river; in this case, an arrow will be put in at the edge of it, as if it was at the end of a chain; now the difference between where it is put in, and where it would have been if no obstacle had come in the way—is the number of links said to be borrowed by the director of the chain, and must be paid as soon as the obstacle is got over,—it is done thus, say the arrow was at 80 instead of 100, therefore the 20 links must be given again after stretching the chain; the next time the director takes up his arrow, and goes

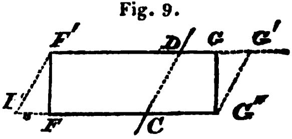
Fig. 8.



forward 20 links, and puts it into the ground, (as this would have been the end of the chain if none had been borrowed,) the chain is pulled on till the handle comes to the arrow, then will all be square again.

In chaining—a house may come in contact with the line as in the annexed figure, & as sides and ends of houses are generally parallel it will be an easy matter to clear such obstacles; produce the back & front of the house equal distances from the points where the line cuts it, as at C D, produce C F equal to D F′, and if any part of the out-houses come in the way, produce D G″ equal to C G″, which will be the same as the other side; now it is evident that the line I F′ is equal to C D, and G′ G″ is also equal to C D, which is the distance through the house. If the distance from C to G″ is 60 links, then must the line D G be produced to 60, which will be at G′ (the above only answers where a point is marked on both sides of the house in line.) From this simple case alone, it is evident that many obstacles may be passed with the chain alone, as making calculations in the field are to be avoided as much as possible.

Fig. 9.

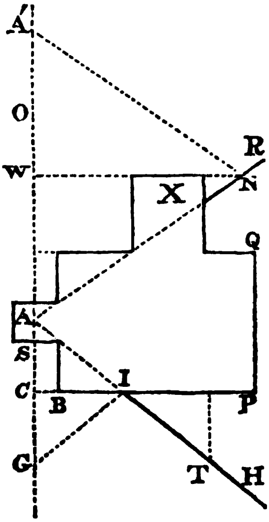


As churches should always be used as trigonometrical stations, (where they are to be had) I will endeavour to explain how measurements may be completed to them. Churches or other buildings when used as stations, should always be measured to, and not from, to prevent errors from being carried through the line, as sometimes it may occur that the true distance is not got from the point used as a station.

Figure 10 will represent the plan of a Church, the centre of its tower being used as a station; the lines H A and IA, are different lines supposed to be running from distant stations into the point A. The dotted lines in the figure will shew the different modes of coming at the distance from the outside of the wall to the point A. I shall first take the line H A as an example.

When the chain reaches the wall at the point I, from which to the point B is measured and found to be 60, and finding that the tower is 30 links square, having a recess of 20, the calculation will be thus, I B + B C = 75 for the base of the right angled triangle, then C S + S A = 35 for the perpendicular, but as this requires a calculation to find the hypothenuse I A, I will shew how it can be got without, by construction on the ground; produce C A, a distance equal to itself, which is 35 links, and from this point G to the point I, will be equal to the line I A, or if on the line I P, a distance is measured equal to I C, and a perpendicular raised to meet the line H A at T, then will the line T I be equal to the line I A. On the other side the line N A can be found by construction; produce the wall X till it meets the line O A, then measure a distance on the line O A from the point W, a length equal to A W, which will be to A′, then will A′ N be equal to A N. Good objects should never be given up for the sake of a little trouble.

Fig. 10.



The principal advantage gained by using Churches or other buildings for trigonometrical stations, is that they are much easier seen, and when the tower is flat, observations can be made from it. These kind of stations require little or no description to find them, as others do, for whenever a station is used in the country, it should be chained to from fixed objects near it, such as bridges, angles of buildings, and other well-defined and conspicuous objects. A large stone should be sunk 3 or 4 feet under ground, with a hole in it to mark the centre, (and it also serves to keep the pole in a vertical position, this is done by driving a round piece of iron into the bottom of the pole, which is inserted into the hole.) These stations may be found at any future period. The whole of the triangulation of the original survey may be made good without fixing fresh points.

Whenever the hypothenuse of a triangle is chained for the purpose of clearing obstacles, &c. instead of squaring the two sides and extracting the square root of their difference, for the other side, it will be easier found by the following rule. Multiply the sum of the two given sides by their difference, the square root of which product will be the side required.

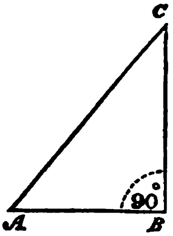
EXAMPLE.

Let the annexed figure represent a triangle, the side A B 30, and A C 50, and it is required to find the side B C.

50 + 30 = 80
50 — 30 = 20

80 × 20 = 1600, the square root of which is 40, the side required.

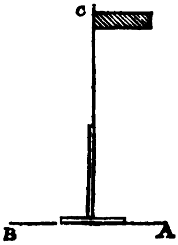
Fig. 11.



In all cases this will answer where A C is one of the two given sides.

When boundaries or other objects are offsetted; care must be taken to prevent their being twisted out of place, and the Surveyor must either use an offset staff, (which is generally ten links long) or something instead to raise the offsets (when they exceed one chain) perpendicular; as the T headed cross is generally used for a levelling staff, it will be found to answer well for the above purpose, the annexed Figure represents the Levelling Cross, as applied in raising perpendiculars. The line A B is the chain stretched on the ground, & the point C the object intended to be measured to. A line may be marked in the centre of the long leg of the cross. This has been fairly tried, and proved by the Theodolite. The difference between them was only from 1 to 1¼ links, in 3 chains length. As

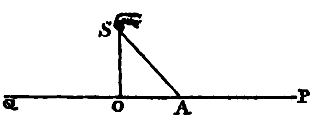
Fig. 12.



I before stated, that it would not be safe to use offsets so long unless sufficiently checked, the cross will answer the double purpose of levelling, and as a cross staff for raising perpendiculars with.

There are various ways of raising perpendiculars by the chain, but the following has been found to be the simplest and most accurate. For instance, let it be required to raise a perpendicular at the point O, as in the annexed Figure.

Fig. 13.

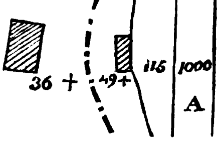


On the line P Q an arrow will be put through the 80 or 20 link ring, fastening the same into the ground at the point O; place a second at 40 links from the same towards A, bring the end of the chain to this point, and direct your assistant to take the 50-link brass in his hand, and mov outwards, then when both portions of the chain are pulled tau to S, the point S will be perpendicular to the point Q.

The above will be found much better than describing arcs on th ground with the chain, or any other flexible instrument.

In chaining lines, and at the same time taking in houses, boundaries, &c. the surveyor should study to keep his book as clear an intelligible as possible, as it does not always follow that the perso surveying should lay down the work. All authors on Surveyin recommend, that when more than one object is noted, on an offset the sign plus should be used, as in figure 14; in short all those wh have written on the subject, have never considered the labour laying such surveying on paper.

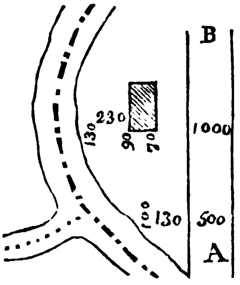
Fig. 14.



When your scale is laid on the line for plotting, as in the annexed figure, you must first mark a point at 115, then add the next number, which is 49, making 164 for another point, add the next number 36 to 164, which makes 200, the total length of the offset, and so on with all offsets of this nature.

Now it is evident that, laying aside the trouble of adding a the offsets together, the plan is ridiculous; for every distance noted is a point for starting afresh with the chain, and gives much more trouble both to the plan-drawer, and the person calculating for the area, as all the different lengths must be added before the length of the whole offset can be ascertained.

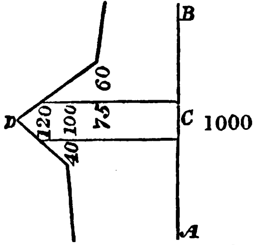
Fig. 15.



The annexed Figure will explain the method of booking offsets; when there are more than one number to be set down on the same offset, for instance, on the line A B, at 1000 an offset is taken to the center of a river; at 70 it cuts one side of a house, and at 90 the other, then at 130 it cuts the nearest bank of the river, and 230 is the whole length of the offset to the boundary. All these numbers can be laid off with ease from a scale; and also if either bank formed the boundary of the ground for content, the offset is at once found for calculation, without further trouble.

In offsetting a very crooked boundary, every advantage should be taken to lessen the labour, by fixing three portions of it at once,* (as in the annexed figure.) Offset the most distant point, from which take the offsets right and left, but bear in mind that it will be much less trouble for the plotter, or person making the calculations, that you should enter the offsets in your book as if they were taken in regular succession; for instance, on the line A B an offset is taken to the boundary, and at 75 on the same line, there is one to the left of 60, and at 100 there is one to the right of 40, and the length to D 120.

Fig. 16.



To book them as if done in succession, will be thus: as the offset at 40 being on the left of the line it comes first, therefore 40 must be taken from 1000, which will give the point where an offset is to be raised of 100 links, and again, 60 (being the offset on the right) must be added to 1000, which will make 1060, at this point the offset will be 75, as in the annexed figure.

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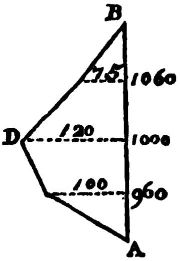
* There are some cases that will not admit of the boundary being offsetted in any other way that as Fig. 16.

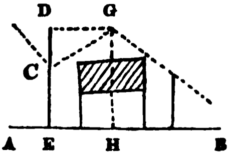
Fig. 17.



It will occur in offsetting a boundary that an obstacle comes between the chain and the point, where an offset is required, such as a house, rising ground, &c.

The annexed figure will explain the method of taking the offset so situated. For instance, a house comes between the point G on the boundary and the chain line A B. Now as the point G could not be seen at H, the offset taken at E to the point C can be produced till an offset can be got to the point G; when the work may either be booked as taken, or entered as if it had been chained from the point H, which is done as follows: to the distance at E add the length D G, which gives the point H, and the length of the offset will be equal to E D.

Fig. 18.



All boundaries should be clearly defined in the book; if a stream, (and the centre measured to as a boundary) it should be marked C S; if either side is measured to as the boundary, then its characteristic must be marked on the side of the fence that the offsets are measured to, and at the same time note in the book what the boundary is—stone wall, hedge, turf bank, &c., and the fences running from the boundary should be noted; this will serve as a proof that the boundary surveyed is the same as represented in the boundary Surveyor's sketch.

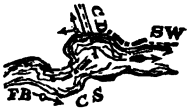
The following will explain how the several descriptions of boundaries are to be entered in the Field Book.

C. R. will be known to be the Centre of Road.
F. B. Face of Bank. C. D. Centre of Drain.
S. W. Stone Wall. R. H. Root of Hedge.
C. S. Centre of Stream. T. D. Top of Ditch.
Md. is a Mound, made where the boundary cannot be traced on the ground.

In parts of Ireland, a ditch is called a sheugh, and the earth obtained from the same is called a ditch. In short, all raised fences are called ditches.

When perambulating the boundaries previous to commencing the survey, good marks should be made at every change, particularly where it changes from centre to side of river.—All these changes are marked by the boundary department, but as all their marks are alike, it has been found necessary to alter them to suit the boundary,—for instance, if it changes from the side to the centre of river, then by making that arm of the mark pointing in the direction of that part to be measured to as the centre into an arrow, as in the annexed figure, the Surveyor will see at once that his offsets must be to the centre, and not to the side. In very crooked rivers it is not an easy matter to find all the changes withont these additional marks, unless the Boundary Surveyor's Sketches are very good, tracings from which are made as a guide to the Surveyor in the field, and are of great service to him in triangulating his work to advantage.

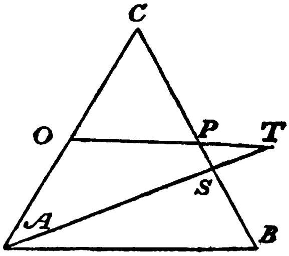
Fig. 19.



The Surveyor in selecting the Stations for the Triangulation of a Parish, should use those lines which will best answer his several boundaries, and not as a matter of course convert his points into well shaped triangles, at the same time another form would have been of more service, and would have enabled him to take in his boundary or roads with less trouble.

In triangulating ground, time is often lost by trying to lay out lines from one fixed point to another; for instance, as in the annexed figure, it was desirable to run the line from A to P, but from obstacles or some other cause, it could not be done without a considerable waste of time; now it is evident if the lines A S and O P were produced till they meet as at T, then will the sum of the triangles A S B and O A T exceed the space

Fig. 20.



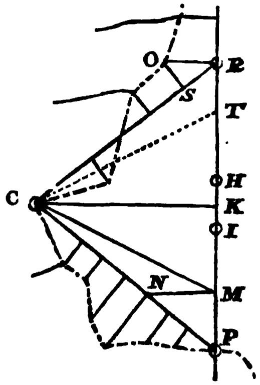
A B P O by the area of the triangle P S T. Every precaution should be taken, that the lines of this small triangle should be checked, and in registering the contents of the several divisions of the great triangle, it may be used as an addative or negative triangle, as the case may be.

The Surveyor should be very cautious in taking in boundary, whether it be Parish, Townland, Bog, Moor, or Lake, (in short, any thing that has to appear in the plan.) When Triangles are made for the above purpose, and not connected with others, such as the edge of an extensive Lake, Sea Coast, &c. then will it be necessary to have a better proof on the boundary being in its true position than depending on the two legs of a Triangle being correctly chained; at the same time an error of two or three chains might have been made, which would affect both the plan and content of the ground considerably. But this being the common method practised by Surveyors, it is presumed they are a body of infallible tradesmen, and that no test is necessary so long as the lines P C and R C will meet.

No such work should be laid down on paper, without the lines P C and R C being directed on some fixed point, or a line such as M C, run to the vertex; and in laying such lines down, if the three do not give one point, it is then certain some of them are wrong, and require re-surveying. When a check line is run, it will be better to let it meet the base obliquely and not nearly at right angles to the same, as an error might have been made there; yet it would fit equally well, and still be twisted out of place, as may be seen by the inspection of the annexed figure.

If the line C K had been used, it is plain that it would fit as well at H, or I, as it would at K; take C K as a radius, and from C as a centre describe an arc, and you will find it coincide with the line P R; the distance of H I; therefore, if

Fig. 21.



any object had been noted on the line, such as a quarry, house, patch of rocks, &c. it would have been out of its true position, but if the line C M had been run, it could only fit at two points M or T, and to avoid an error, by using the wrong point, it will prove if the line is in its proper place, by measuring a short line such as N M.

In forming offset Triangles, attention must be paid to the offsets, otherwise some of them will give a deal of trouble both in registering and finding their areas.

RULES FOR THE CALCULATION OF TRIANGLES AND OTHER FIGURES, OCCURRING IN SURVEYING.

The three sides of a Triangle being given to find the Area.

Rule. Add the three sides together, and take half their sum, from the half sum subtract each side separately, multiply the half sum and the three remainders continually together, and take the square root of the last product for the area of the triangle.

Example. Required the content in square chains of the Triangle, whose sides are 6000, 8500, 9345 Links.

6000	11922·5	11922·5	11922.5
8500	6000	8500	9345
9345	5922·5	3422·5	2577·5
2)23845			

11922·5 Log. = 2·0763674
5922·5 Log. = 1·7725051
3422·5 Log. = 1·5343435
2577·5 Log. = 1·4111987

2)6·7944147

3·3972073 = 2495·785

Answer, 2495·785 Square Chains.

I have made the foregoing calculation by Logarithms, as where Triangles are large, it would never answer to do them by natural numbers. You can see that for 11922·5 the index is only 2, because it is worked in chains, and decimals of a chain, and it is a fixed rule, that the index of a Logarithm is always equal to one less than the number of figures on the left of the decimal point. For example, the index of 12 is 1, that of 120 is 2, and so on; but this is supposing them all to be whole numbers. In looking for the natural number corresponding to a Logarithm it must always have one place of Integers more than the Index represents,—for instance, for a Logarithm whose index is 4, the Integer's will be 5, and for 5 there will be 6 places of whole numbers.

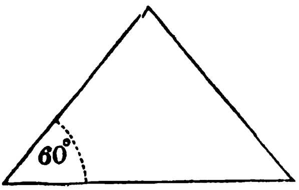
Areas of Triangles are calculated sometimes by the two sides and the contained angle. This will serve as a check when the area is computed by only one person, and answers well in the case of large triangles, as the calculation is much shorter.

Rule. Multiply one of the sides by half the other, and that product by the Log. sine of the contained angle. The last product will be the area of the triangle.

Example. Required the area of a triangle whose two sides are 31323 and 25534, and included angle 60°. Fig. 22.

$$\frac{31323}{2} = 15661{\cdot}5 = \text{Log. } 2{\cdot}1948333$$

25534 do. 2·4071189
Log. sine of con. angle 9·9375306
Sq. Chains 34632·41 = 4·5394828
Or 3463 Ac. 0 Rood, and 38 Perch.



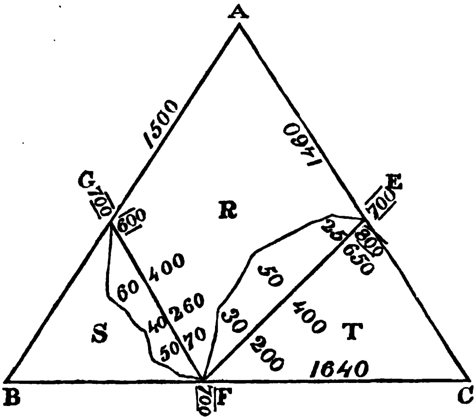
In calculating areas you must take out 2 places of decimals in addition to the number required by the index.

Offsets are calculated either as Triangles or Trapezoids; for instance, the offsets on the line F E, convert it into two of each. The calculation will therefore be as follows—commencing at F 200 × 30 divided by 2 = area of the first triangle, then 200 taken from 400 (being the point where the next offset is taken) leaves 200 for the length of the base of the trapezoid, and 30 + 50 = 80 divided by

2 = 40, which gives the mean breadth of it, which sum × 200 = area of the first trapezoid.

In this manner will all the offsets be calculated whose areas are not computed from paper, but in order to have a proof that the small divisions in a triangle do not exceed, or are less than the large one, it will be necessary to have the whole of the interior divided into triangles,* without leaving a Trapezium, as in the annexed figure, for it is evident there is no proof that the quadrilaterial GAEF is correct. It is taken for granted that the other two triangles are correct, their areas taken from the large triangle, leaves the area of the quadrilateral;—but to ensure accuracy, the trapezium GAEF should be divided into two triangles, by running a line from G to E, which would not only serve to ascertain its true area, but would also have determined the position of E and G.

Fig. 23.



A considerable error might have been made at E, without the person laying down the work being able to discover it, as F E meets the line A C, nearly at right angles; as it may be seen by inspecting the figure, that with a radius of F E, an arc described from F as a centre, will coincide with the line A C, for a considerable distance. Consequently the point E might as well have been at any part of the line that agreed with the arc; and this error would affect division T as well as division R.

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* The Surveyor must bear in mind that unless all the interior lines of the triangle are measured accurately, and errors of the chain taken notice of, (if any) the sum of the lesser divisions can never be equal to that of the larger.

METHOD OF REGISTERING FOR COMPUTING AREAS, FROM ACTUAL MEASUREMENT IN THE FIELD.

I will now explain a very simple method of registering for content, two sheets must be used, one for additives, the other for negatives; (if the survey be extensive,) and whenever any division of ground is began it should be completed, *i. e.* all that is within your triangle, before you commence another.

The 1st column of the Register Sheet contains Letter and No. of Plan or Plot; the 2nd contains the name of the Division, or Subdivision of Land; the 3rd, Triangle, or Trapezium; the 4th is the 1st side of the triangle; the 5th is the second side; the 6th contains the third side; the 7th is the content in square chains; and the 8th is the content in statute acres.

Example of registering the divisions in the Triangle A B C, for the purpose of ascertaining the areas of each separately. Before I proceed I will explain the method of arranging the sides to avoid confusion. In every Triangle make the first and second letter represent the first side; first and third the next, and second and third the last; too much attention cannot be paid to this branch of the survey.

Commencing with division R, I see by the content plot or Field Book that it is neither a Triangle or Trapezoid, therefore I must register the whole Triangle A B C (see Fig. 23,) in the additive sheet, then I register the triangles E C F, and G B F in the negative. I next commence with additive and negative offsets, which will be registered thus: I begin with the line F G, in the column marked 1st side, enter in black ink, the distance between each offset, and in that marked 2nd and 3rd side enter in red the length of the two offsets, but where there is but one (as in this case) to begin with, leave column marked 2nd side blank.

The offsets being set down in red, the person making the calculation will know what they are, as offsets are not calculated by the three sides as triangles, which was before explained. Begin at F, place 70 in the 4th column, 0 in the 5th, and 50 in the 6th, then 70 from 260 leaves 190, for the distance between the 1st and 2nd offsets, this again goes in the 4th column, 50 in the 5th, 40 in the 6th, and so on till all are gone over, then a line will be drawn under all belonging to the division of ground you are making up. In the negative sheet do the same with E and F, as you can see it does not belong to R, then will the difference between their sums be the area of division R; and to register division T, the Triangle C E F, with offsets on the line E F will be its content; and also for division S, the Triangle B F G will be additive with offsets on G F negative. All of these will have been calculated before in finding the content of R, therefore nothing but the letters should be entered in the register, stating what page their contents may be found in.

Form of Content Register Sheets.

ADDITIVES. 1

Plan and Plots.	Division or Subdivision.	Triangle or Trapezium.	1st Side.	2nd Side.	3rd Side.	Content in Square Chains.	Content in Statute Acres.
	R	A B C	1500	1460	1640		
		F	70	—	50		
			190	50	40		
			140	40	60		
		G	200	—	60		

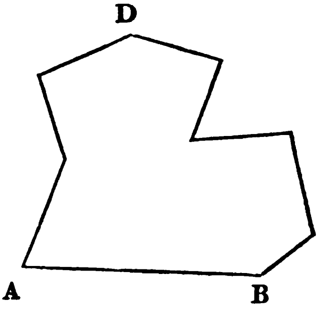
NEGATIVES. 1

Plan and Plots.	Division or Subdivision.	Triangle or Trapezium.	1st Side.	2nd Side	3rd Side.	Content in Square Chains.	Content in Statute Acres.
	R	E C F	700	800	940		
		G B F	700	600	700		
		E	200	—	30		
			200	30	50		
		F	250	50	25		
			150	—	25		

ON EQUALIZING AND COMPUTING AREAS FROM PAPER.

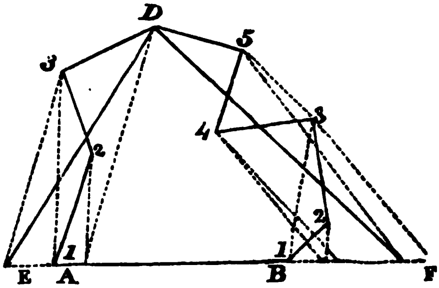
Very small portions of crooked boundary are sometimes reduced to triangles, without registering the offsets separately, but to come at any thing like the truth, each space should be done by two different persons, when convenient; and if found to agree, the mean of their areas may be taken, but in the event of their differing more than one arc in three hundred, a third reduction will be necessary, in order to find which of the first two were right. Larger divisions, such as Glebes, Town Lands, or other sub-divisions of a country may be reduced also, and will serve as a check on the Register; but as the calculations are made from measurements taken from paper, mistakes often occur in measureing the distances; to remedy this defect the three sides should be used as three different basis, each time measuring a new perpendicular. In this way you come nearer the truth than any other manner, except by the calculation of actual measurements on the ground. Some authors have recommended that surveys should be plotted on stiff card cut out and weighed, by which means the areas may be calculated. Now paper is a substance so well known to expand and contract with the change of the atmosphere, that those who recommend this mode cannot have practised surveying with any degree of accuracy.

Fig. 24.



The following problems will be found useful to the Surveyor in reducing irregular Polygons to Triangles.—Suppose the annexed figure is to be converted into a Triangle. First I commence by choosing a point on the figure that will become the vertex of the same, I next number the points from each extremity of the figure towards the point D, which will be on one side 1, 2, and 3, the other will be 1, 2, 3, 4, and 5. I find the line A B will form a good base to reduce the figure upon. Therefore it will be produced to right and left, place the edge of a parallel ruler to the point marked one, or A. (on the left) and the point 3, then move the parallel ruler up or down till it cuts the point marked 2, and note where it cuts the line A B produced, this will be a new number 1, the original one will be marked that it may not be used a second time.—From the new number 1, to the point D, apply the edge of the ruler and move it up or down till it cuts the point 3, note also where this meets the line A B, this will be the extremity of your triangle on this side of your figure; as E. Draw a line from the last found point E to the point D, which if properly done will cut as much off the figure to be reduced as it has taken in.—Proceed with the other portion of the figure in exactly the same manner, commencing with B, and 3, move up or down till it cuts 2, noting where the same line meets A B produced, which will be a new number 1, from it to the point marked 4, place the edge of a ruler and move up or down till it cuts the point 3, and note also when the line meets A and B produced, which will be another number 1, move from which to the point 5, move as before till it agrees with the point 4; again note where this meets the line A B produced, from which point to the point D apply the ruler, and move up or down as before till it coincides with the point 5, noting where it meets the line A B produced, as F, this will be the other extremity of your triangle, from which draw a line to the point D, this will be the other side of the triangle required. You have now converted the irregular Nonagon into a Triangle, equal in area to that figure. The dotted lines shew the construction.

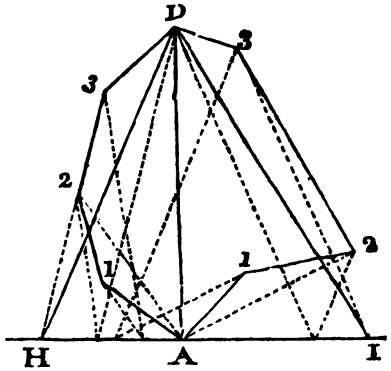
Fig. 25.



A curve lined figure may be reduced in like manner, and the the greater number of points on the curve, the more accurate will the reduction be.

The annexed figure will shew how the same may be converted into a triangle without making one of its sides any portion of the base of the triangle; the dotted lines will show its construction; in reducing figures in this manner, the line H I tangent to the point A, need not be confined to any particular direction, but must be regulated by the form of the figure and the size of the paper which you are working upon. In this case you can perceive that the line H I stands nearly at right angles to the line A D, which divides the figure into two parts, each of which must undergo the same process of reduction as the last figure. It is evident that as both the triangles are upon the same base, they may be calculated as one. The foregoing method of reducing irregular figures to triangles, is founded on the well-known theorem, that triangles on the same base, and between the same parallels are equal to each other.

Fig. 26.



In registering areas computed from paper, their content is registered without entering lengths; by the letter at each end of the space being entered in the second column of content register sheet. No distances must appear in the registers but those actually measured on the ground, or derived from trigonometrical calculation. It is to be remarked, before the register of a parish or other division is commenced, a letter must be placed on the content plot, at every place where a boundary is crossed by a line, in order that any of the spaces may be the easier found. The content Field Book should, in every case, correspond with the content plot. The register sheet should be compared, to see that nothing has been omitted, as sometimes it occurs that triangles or other spaces are registered as additives in place of negatives. In making these comparisons, a plotting scale should be used; to try the lengths of the lines, as sometimes in registering, mistakes occur, by the figures being read in an inverted position. After the several areas are computed, should the summation of the townlands or other sub-divisions exceed the general or total area of the parish, or other large division more than one acre in one thousand, it will be necessary to look over the registers and content plot in order to find out the error. Should it appear that the content plot and Field Book are identical, it is evident then that the error is in consequence of bad chaining; in filling up the interior of the triangles.

Small crooked portions of boundary may be equalized with a slip of clear horn, through it you will see where a line may be drawn that will cut off as much of the figure as it will take in. A small piece of whalebone and a fine hair in the form of a drill bow, will also answer well for equalizing portions of crooked boundary.

If in practice a circular figure is measured, it will be better to compute its area in the regular way, as it would require so many points on the curve to reduce it to any thing like a right lined figure, as for hedges or rivers being segments, and calculated either by equi-distant offsets, or by the sector, would be a waste of time, and when done would be but a guess, as correct measurement of circular figures is impractical—if the offsets are numerous they will give the content very near the truth, as there are few hedges that will not agree in short portions with a straight line.

In ornamental ground there may be circular figures, if so, find their diameters, and should their content be required, the following Rule is inserted as being the most simple of many given for finding the area of a circle, viz.

Square the diameter, and multiply that sum by ·7854, the product will be the area.

The decimal ·7854 is the area of a circle whose diameter is one.

To find the area of an Ellipse.

Rule.—Multiply the transverse by the conjugate axis, that sum by ·7854 will be the area, nearly.

LEVELLING BY VERTICAL ANGLES WITH THE THEODOLITE AND CHAIN.

The relative heights of objects may be obtained in a very expeditious and simple manner, by the Theodolite. When a line is to be levelled, the instrument is set up at one extremity of it; two assistants are required, each of whom must have a rod (of the same height as the instrument) with a T head, one of them moves on to the point where a level is wanted, then direct the telescope on the top of his rod, clamp, and level it, and read the error, (as unless the instrument is in true adjustment in all its parts, and accurately levelled at every station, there will be an error) and enter it in the 3rd column of the Levelling Field Book, (a form of which is given.) Then read the vertical angle to the object, and set it down in the 4th column. If the angle is elevation, Zero will stand to the left of the arrow on the vertical arch, if depression to the right. The 5th column contains the No. of links to be subtracted for each chain's length, as shewn on the back of the vertical arch, marked " Difference between Hypothenuse and Base." This scale is used only when the ground between two objects is of one uniform slope. In the 6th column should be entered a few horizontal angles, taken to distant objects, to serve as a check on the chain survey, together with any remarks that may be thought necessary.

To ensure accuracy in the determination of the difference of level between objects, reciprocal angles (*i. e.* back and forward angles) must be taken, which does away with the necessity of making allowance for curvature, &c. The stations used for levelling in this manner, should never be distant from each other more than 50 or 60 chains. The observations taken from one extreme station to another, should always be to the ground at the base of them as in taking tops of poles there will be much more trouble in finding their heights than if the ground had been taken, as they may be of different lengths, and out of the vertical position, when observed.

The heights of many fixed and remarkable objects, such as houses, mills, bridges, &c. may be determined in the course of levelling, as follows:—set up the instrument at any of your stations, level it very accurately, turn it round, and note in your Field Book any object that is cut by the line of collimation. Such objects will consequently be the height of the instrument above the place where it stands. At the same time the angle contained between any such objects and the station, should be taken to ensure their position.

The height of objects whose distance is known, may be fixed in this manner, for a distance of two or three miles around the place of observation, taking into consideration always the allowance for curvature and refraction.

Form of Levelling Field Book.

Letter from.	Letter to.	Telescope Levelled.	Apparent Elevation or Depression.	Distances on the Line.	REMARKS.
A	*a*	E 4′	Dep. 10° 20′	3560	A. Aughrim.
a	A	E 1′	Elev. 10° 24′		*a*. South side of road leading from Dublin to Belfast.
	b	0′	Elev. 7° 35′	6245	
b	*a*	D 4′	Dep. 7° 39′		*b*. Summit of a hill southside of which passes the above road.
	B	D 2′	Elev. 11° 37′	8245	
B	*b*	D 1′	Dep. 11° 40″		B. Blackhill.

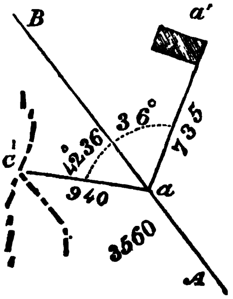
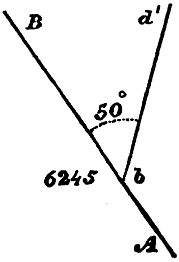
In levelling, numerous stations may be saved by using a rod from 15 to 20 feet long; as it happens sometimes that a wall or other obstacle comes in the way. The difference of level between any such stations, may be obtained by the common method of levelling, and entered in the Levelling Field Book, as if it had read 0, merely noting the difference of level between them; or when practicable, to measure the distance between two such objects, the angle of elevation or depression may be taken to the top of the pole, but in this case, as you cannot obtain a reciprocal angle, the height of the pole, as well as that of the instrument must be noted in the column of remarks.

Measuring lines and reducing them by the scale on the instrument, is attended with a deal of trouble; as at every distance an object is noted, a calculation is necessary in order to find its true position on the line. When side levels are taken (*i. e.* objects levelled to on either side of the main line,) they should not be entered in the same page, as sometimes they create confusion both in calculating their relative heights, and laying them down on paper; such should occupy a separate part of the book. The alphabet used for the levels on the principal line, should differ from those used for the side levels,—for instance, if on a line the stations at which the different levels are taken, are marked *a. b. c.* &c. the side levels taken from the same stations should be distinguished by the letter of that station, and another letter with some characteristic mark over it, such as *a'*, *b'*, *c'*, &c.

When objects are levelled to, and not on the line, a small diagram (as in the next page,) should be entered in the column of remarks, pointing out their relative position with regard to any station on the line.

If the 5th. column of the Field Book, is not used for the purpose mentioned before, the distance of each levelling station from the commencement of the principal line may be entered in it, one of them subtracted from the other will consequently give the distance between these two stations, and this number will be entered in the 2nd. column of the Register sheet. (See page 37 and 40.)

Side Levels on the Line A B.

Letter from	Letter to	Teles. Levelled	Apparent Elevation or Depression.	Distances on the line.	REMARKS.
a	a′	o	Dep. 7° 34′	735	 B a′ 36° 4236 735 c′ 940 a 3560 A
a″	a	3′Dep.	Elev. 7° 31′		
a	c’	1′Dep.	Dep. 10° 36′	940 a	*a′* is the East angle of Church ruins. Glebe House, 276°10′ black hill; B 360; *c′* meeting of three parishes 317° 24 distant from *a* 940.
c’	a	5′Dep.	Elev. 10° 31′		
b	d′	o	o o		*d′* Eaves of Chapel at Ballyrogan, Eight feet above the base. 
			Theodolite 4·5 high.		

The method of registering vertical angles to prepare them for calculation will be as follows.

Form of Register Sheet for the Calculation of Vertical Distances.

PLAN.	Measured Horizontal Distances.	Elevation or Depression	Calculation of Vertical Distances.	Relative Alt. in Feet.	Altitude above L. water mark.	REMARKS.
B 3	A 3560 *a*	D 10°23'30″	9·8195439 3·5514500 9·2633613 2·6343552	430·87	1347·5 916·6	A Aughrim *a* S. Side of Road leading from Dublin to Belfast.
	a 2685 *b*	E 7° 35'	9·8195439 3·4289443 9·1242839 2·3727722	235·92	1152·5	*b* Summit of a hill, S. side of which passes the above road A 1347·5 B 1424·6 Diff. of level. } 77·1
	b 2000 B	E 11° 39'	9·8195439 3·3010300 9·3142468 2·4348207	272·15	1424·6	B Blackhill.

One example, in illustration of the above form will suffice. In the first column will be found the No. or letter of the plan; in the second, the measured horizontal distances, with the letters of the stations between them; in the third, the mean of the reciprocal angles, which are obtained as follows.

The first observed angle from A to *a* was 10° 20' of depression, and the error of the instrument 4' of elevation, which being added to the angle of depression = 10° 24'. (The error is the reading of the vertical arch, when the telescope is levelled, if elevation, and the observation to an object is depression, then it must be added, and *vice versa.*). The back angle from *a* to A was 10° 24′ of elevation, and the error of the instrument 1′ of elevation, which being subtracted from the angle of elevation, leaves 10° 23′. The mean of which two angles will be found as follows.

10° 24′ of Depression.
10 23 of Elevation.
20 47
2 = 10° 23′ 30″ as the true angle to be entered for calculation.

In the fourth column is entered the arithmetical complement of $\frac{66}{100}$ths. the log. of the measured distance, and the log. tangent of the angle, all of which are added together, and the natural number answering to their sum will be the relative altitude in feet, which is entered in the 5th. column. If the horizontal distance is measured, the log. tan. is used, but if the slope or hypothenuse is chained, then the log. sine will be taken.

The sixth contains the height of the object above low-water mark, which is always entered in red, when deduced from the great Trigonometrical survey, or by levelling from the sea; and under the head of remarks, a short description of any of the remarkable stations levelled to, is entered.

The arithmetical complement of $\frac{66}{100}$ths. is one which is generally printed in the forms of such registers, as distances being generally measured in links, by the addition of it, the height is obtained in feet.

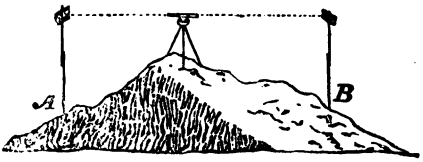
When Trigonometrical Stations are situated near the sea, they should be levelled to from low water mark, (at Spring Tides,) which will serve as a check on the altitudes of Stations, which might have been previously found and carried through a considerable extent of country. The simplest mode of doing which will be as follows.

LEVELLING WITH THE THEODOLITE AS A SPIRIT LEVEL.

Set up the Instrument at any point not more than the height of itself, below the station you are working from; direct the Telescope towards your assistant, who will stand at the base of the station,and note where the cross wires cut the rod, and set that down in column marked back stations, then turn the telescope round to your second assistant, causing him to move up or down his rod till it agrees with the cross wires as before, and whatever the height may be set it down in the column of forward stations. In this manner proceed till you arrive at the point you wish to level to.—Then will the back stations be added up, and also the forward ones, and should any distance exceed 1000 feet, the corrections will be taken from the heights found, as may be seen by the following form, here the Table for Curvature and Refraction, given in page 46, will be found useful, where the distance between the levelling points are of such a length as to require correction.

By levelling in this manner, no notice is taken of the height of the instrument, as may be seen by the annexed figure.

Fig. 27.



If the tops of the levelling rods A and B are brought to the same level, should B be 11·9, and A 17·7, then will the difference of level between the two points be 5·8, which is obtained without paying any attention to the height of the instrument. The rods used for levelling should be divided into feet and tenths.

Example of booking a series of levels between two Stations, for instance, a Trigonometrical station, and a stake driven in at low water mark.

The distance (where I first placed my instrument) to the Station is found to be 1320 feet, and the height 1·53 feet, here I find the correction for that distance to be ·03 feet. I find then my forward Station is distant 2000, height 13·25, correction ·07; in this manner I would continue till I arrived at low water mark. Then all the heights of the back, and forward Stations will be added up, and the corrections taken from the height so found, then will the difference between the back and forward Stations be equal to the difference of level between the Station and low water mark, as may be seen by the form below.

REMARKS	Back Station.			Instrument.	Forw. Station.			REMARKS.
	Cor.	Hei.	Dis		Dis.	Hei.	Cor.	
A								
Trig. Station at Bann Mouth.	·03	1·53	1320	A B	2000	13 25	·07	B Roadway of Bridge.
	·02	·25	1000	C D	2440	17·75	·14	
	00	·21	130	E F	620	18 25	00	Low W. Mark
								F S 49.02
	05	1·99				49·23	·25	B S 1·94
		·05				·2 1		
								47·08
		1.94				49 02		Diff. of level between Station & L. W. Mark

INSTRUCTIONS FOR CORRECTING THE ALTITUDES OF OBJECTS WHEN CURVATURE AND REFRACTION ARE TAKEN INTO ACCOUNT.

Terrestrial Refraction is that which causes objects to appear higher than they really are, which vary considerably with the different states of the weather, and atmosphère. Curvature, is the difference between the true and apparent level, which is equal to the square of the distance between the places, divided by the earth's diameter; consequently, it is always proportional to the square of the distance.

The quantity of Refraction is estimated in various ways, some allow $\frac{1}{10}$th. of the distance of the object observed, expressed in degrees of a great circle; others $\frac{1}{11}$th, $\frac{1}{12}$th, and $\frac{1}{14}$th.; but $\frac{1}{12}$th. seems to have been used by the Officers making the Trigonometrical Survey of England; $\frac{1}{9}$th. having been found to be the greatest quantity, and $\frac{1}{15}$th. the least.

The following simple method will be found sufficiently accurate for reducing the angle of observation.

Rule. Divide $\frac{1}{12}$th. of the distance between the objects by 101·28 (the number of Feet answering to one second of a degree) the quotient will be the number of seconds contained in that distance, which must be taken from the observed angle.

Example.

The distance being 47520 feet (9 miles) and the angle of Altitude 1° 40′ 20″, the correction of which will be as follows:

47520 ÷ 12 = 3960 ÷ 101·28 = 39′ Number of Seconds.

Observed angle .... 1° 40′ 20″
Correction for refraction.. 0 0 39

Corrected angle 1 39 41

The allowance of Refraction may be obtained by taking $\frac{1}{9}$th. of the square of the distance in miles for the number of feet, for instance, a distance of 9 miles will give 9 feet, because 9 × 9 = 81 ÷ 9 = 9.

With the angle reduced, it will be found the same, as may be seen by the following calculations.

Curvature is obtained as follows, two-thirds of the square of the distance in miles will give the number of feet for curvature, one sixth of which will also give refraction, by this Rule the corrections for curvature and Refraction are made in the secondary Triangulation of Ireland. It must be borne in mind that Refraction is always a minus quantity, curvature may be either when the object observed is elevated, curvature is added to the height found, when depressed it is taken from it.

Let A B represent two stations distant from each other 47520 feet, (9 miles) with observed and corrected angle as above.

Calculation with corrected angle.

Log. of Distance A B 4·6768764
Log. Tangent of Corrected angle 8·4624704

3·1393468 = 1378·3
Curvature 54

Total Height 1432·3

Calculation with angle as observed.

A B 4·6768764
8·4652947

3·1421711 = 1387·3
Tabular correction for 9 miles 45

Total Height 1432·3

We shall now suppose the point B to be below the point A, the calculation will be thus:—difference of height 940 feet, correction for curvature and refraction 5 feet, and the height of the instrument, which is 4 feet, (that is when the observation is taken to the ground) the correction will stand thus:—the distance between them being considered 3 miles. Height found 940.

Tabular correction for 3 miles = 5, and Height of Instrument = 4, making 9, which subtracted from 940, leaves 931, being the total height of A above B.

I shall introduce one more example to shew the utility of the preceding rule.

Suppose the distance from an observer to the summit of a mountain be 32 miles, (the summit appearing in the horizon) required the difference of level between the two places.

The square of 32 = 1024, two-thirds of which is 682·66, one-sixth of the same = 113·77, that taken from 682·66 = 568·89; consequently the height of the mountain above the observer is 569 feet, nearly.

Several authors have introduced tables, purporting to calculate the heights of distant objects; for instance, in Burr's Treatise on Practical Surveying, (see page 171) for a distance of 12 miles, he makes the difference between the true and apparent level 96 feet, in this case we differ 16 feet; who, although he speaks of a correction being necessary for refraction, has not used it in his tables.

TABLE OF CURVATURE AND REFRACTION FROM 1000 TO 63360 FEET, BEING 12 MILES.

Distance in feet.	Correction in feet and decimals for Curvature and Refraction.	Miles reduced to Feet.		Correction in feet and decimals for Curvature and Refraction.
		Miles	Feet.	
1000	·02	4	21120	8·89
1320	·03	4¼	22440	10·03
1600	·05	4½	23760	11·25
2000	·07	4¾	25080	12·53
2640	·14	5	26400	13·89
3000	·18	5¼	27720	15·31
3960	·31	5½	29040	16·80
4500	·40	5¾	30360	18·36
5280	·55	6	31680	20·0
6000	·71	6¼	33100	21·70
6600	·86	6½	34320	23·47
7000	·96	6¾	35640	25·31
7920	1·25	7	36960	27·22
8500	1·42	7¼	38280	29·20
9240	1·70	7½	39600	31·25
10560	2·22	7¾	40920	33·36
11000	2·40	8	42240	35·55
11880	2·81	8¼	43560	37·81
13200	3·47	8½	44880	40·14
14520	4·20	8¾	46200	42·53
15840	5·0	9	47520	45·0
16000	5·10	9¼	48840	47·53
16500	5·86	9½	50160	50·14
17160	5·40	9¾	51480	52·81
18480	6·80	10	52800	55·55
19000	7·16	11	58080	67·22
19800	7·81	12	63360	80·00

METHOD OF REDUCING FEET TO LINKS, AND *vice versa.*

As distances are always obtained in feet in consequence of the principal triangulation being so, it will be necessary to reduce them to links, to suit the interior chaining, and to facilitate the calculation.

The reduction may be as follows: divide the feet by ·66, the quotient will be the number of links.

EXAMPLE.

In 5280 feet, how many links?

·66)5280.00(8000 Links. *Answer.*
528
——
000

Mr. Croker says, multiply the feet by 1·515, a link and its decimal contained in one foot, now this is all a hoax, here follows the proof. In page 167, he says the distance from Mendip to Long Knoll is 61963·5 feet, and from Mendip to Westburry Down is 92954 feet, the former he makes 93874·7 links, and the latter 140825·3, instead of 93874·7 it should be 93884·09 a difference of 9·39, and instead of 140825·3 it should be 140839·39, difference 14·09. This would effect the area of the triangle considerably. A triangle calculated with the above corrected sides, and contained angle, will produce an area different from that of Mr. Croker's of 8 acres and 17 perches; however small this difference may appear to be, it is sufficient to convince any one of the absurdity of reducing feet to links by the method alluded to.

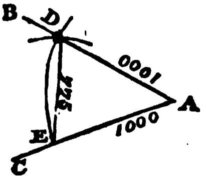
In reducing links to feet, multiply the number of links by ·66, and the product will be the number of feet.

MEASURING AN ANGLE BY A CHAIN OR TAPE.

As it is not always convenient to carry a Theodolite when erecting Stations; whenever the place of a Station is decided upon, the directions of two or three distant Stations should be got, in order to fix its position, which may be done as follows.

Example 1 will explain the method of finding the contained angle when measured by the chain. This will be found useful in many cases, particularly in finding your position on a diagram when employed in erecting Trigonometrical Stations. It will answer sufficiently near for a rough diagram of the points to assist the triangulation. Measure 1000 from A towards C, and a like distance from A towards B, then measure the distance between these as DE, and in this case it is found to be 775, now to find the contained angle it will be as follows.

Fig. 28.



Example 1. To find the angle E A D, the natural sine of half the distance D E will give half the included angle, N. Sine of 775 divided by 2 $= 387{\cdot}5 = 22^\circ\ 48'$ multiplied by 2 $= 45^\circ\ 36'$, which is the required angle.

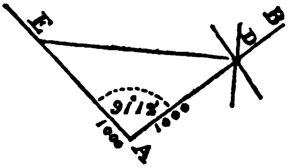
Again, if the angle was given, and the two measured distances, to find the line D E. It is the reverse of the last and will be found thus, double the N. Sine of half the contained angle will be equal to the line D E.

$$\frac{45^\circ\ 36'}{2} = 22^\circ\ 48' \text{ N. Sine of which is } 387{\cdot}5 \times 2 = 775.$$

This Problem will be very servicable for getting the direction of the side of a triangle, when the paper will not admit of the whole of it being protracted, as lines are easier laid down than angles.

Example.—Suppose the contained angle was 91° 12′ (as in the annexed figure) and it is required to protract the line A B, it is done thus: with a radius of 100 or 1000; from the point A describe an arc outwards, cutting the line E, look in the table of natural sines corresponding to 45° 36′, (which is half the contained angle), it gives 714, that multiplied by 2 will be equal to 1428, then with that as a radius from the point E, describe an arc intersecting the former, which will give the point D, then a line drawn from A through the point of intersection last found, will give the direction of B.

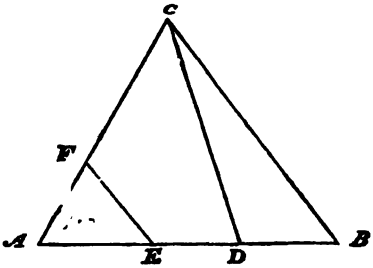
Fig. 29.



A FEW SIMPLE RULES FOR DETERMINING THE UNKNOWN PARTS OF A TRIANGLE.

When Triangles are large it will be necessary to calculate the lengths of some of the interior lines, as a check on the chaining, without any additional angles being observed than were first obtained in the great Triangle, providing the surveyor makes it a rule, that his lines run from one side to the other, or from one side to the vertex, as in the annexed figure.

Fig. 30.



1st. Having two sides and contained angle given, to find the third side.

Rule. $BC = \sqrt{(AB^2 + AC^2 - {}^{*}2AB \times AC \text{ Cos. } A.)}$ which is simply this,

Take twice the rectangle of the two sides, and the cosine of the contained angle, from the sum of the squares of the two sides, (if the angle is less than 90°, but if greater to be added) and the square root of the product will be the side required.

M. Cagnoli says he considers this rather troublesome in practice, for which reason he has given another formulæ, but for all that is required here, this will answer the end, and will be found sufficiently simple, if compared with the round-about rule generally practised; (where the unknown angles have to be found) it must be admitted, that as a practical rule Cagnoli's has the advantage.

Example. The side A B 313·23, A C 255·35 chains, and the included angle 60° given, to find the side B C.

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* The last term is positive when the angle is obtuse.

Log. A B	2·4958634	× 2 = 4·9917268 =	98113·0
A C	2·4071359	× 2 = 4·8142718 =	65203·6
of 2	0·3010300		
Cos. of 60°	9·6989700		163316·6
	4·9029993		79983·3
			83333·3

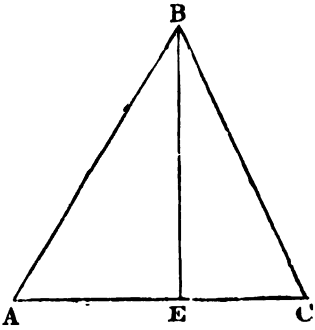
Square Root of which is 288·675, for the side B C.

The advantage of finding the third side in this manner is obvious, as the rectangle of the two sides is obtained by adding the Log. of 2, which may be considered as a constant, and ought to be remembered. It is 0·3010300. After the Logs. of the two sides are obtained, there will be no necessity for transcribing them, as they may be multiplied by two in the order for calculation as in the preceding form. The side will be obtained more accurately, by taking out a few places of decimals in addition to the number of intigers required by the Index.

The Surveyor may have given him a Triangle for the purpose of filling up; if the ground is unfavourable, it will be an advantage to know the angles formed by the three sides, as the stations may not be seen from each other, or obstacles may prevent his seeing points he may wish to run to, in short the problem may be applied to many purposes.

Three sides of a Triangle given, to find the point where a perpendicular let fall from the vertex will meet the base.

Fig. 31.



Example. Let the annexed figure represent a Triangle, and we wish to know how the line A C will be divided, by a perpendicular falling from the angle A B C. As the Base A C is to the sum of the other two sides A B and C B, so is the difference of those sides to the difference between the segments A E and E C; half of this difference will be added to half of A C, which will give A E; and half the same difference taken from half of A C will give the lesser segment E C.

If we suppose the sides to be as follows, C B 12000, A C 11500, and A B 12800, the Base A C will be divided as follows. As 11500 : 24800 : : 800 to the difference between the segments A E and E C, which is 1725·2 ÷ 2 = 862·6, which being added to half the side A C will give the greater segment. Thus 11500 ÷ 2 = 5750 + 862·6 = 6612·6, and for the less segment it will be 5750 — 862·6 = 4887·4, the sum of the two segments will be equal to the 11500.

The greatest segment will always be next the longest side.

Thus finding the point E, the Triangle is converted into two right-angled Triangles, consequently all the other angles will be found with ease.

For instance, if we require the angle E B C with sides as given above, it will be found thus; Add the Sine of the given angle (90°) to the Log. of the side adjoining that angle, to which sum add the Arith. Co. of the side opposite the given angle, the product will be the sine of the angle opposite the adjoining side, the calculation for which will be as follows.

C E = 4887·4 Log. of C E + sine of C E B = 13·6890779
C B = 12000. Arith. Comp. of C B = 5·9208188

Log. Sine of the angle C B E 9·6098967

which is 24° 2′, that taken from 90° will leave the angle B C E, 65° 58′.

By this means will all the angles be found if necessary in any Triangle, and should the unknown side B E be required, it may be found as follows.

Example. Let B C and angle B C E be given to find E B, Add the Log. sine of the angle C to the Log. of the given side C B, the sum abating Rad. will be the required side E B.

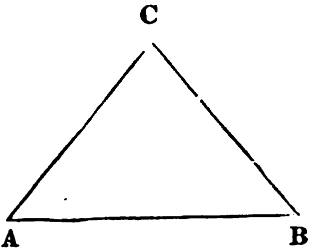
But if the Base C E be given, use the Tangent of the given angle C instead of the sine.

Having one side and the angles given to find the other two sides.

Rule. Add together the Logs. of the given side, the sine of the angle opposite to the side required, and from which take the sine of the angle opposite to the given side, the remainder will be the Log. of the side required; ~~and for every Arith. Compt. so added 10 must be subtracted from the sum of the indices.~~

In calculating the lengths of lines where the angles are oblique, time will be saved if the Arith. Com. of the sine of the angle opposite to the given side, is added instead of subtracting its Log. Sine from the other two, then the Tables need not be consulted so often, as may be seen by the following example.

Fig. 32.



Example. In the Triangle A B C having the side A B 32460, and angles A C B 34° 12′ 40″ C B A 73° 25′ 50″ and C A B 72° 21′ 30″, to find the sides A C and B C.

Log. of A B = 4·5113485
Sine C A B = 9·9790795
Arith. Compt. A C B = 0·2500754

Log. C B 4·7405034 = 55017·8
Sine A B C 9·9815807
Log. A C 4·7430046 = 55335·6

By the foregoing calculation it will be seen that after adding together the Logarithms of the side A B, sine of the angle C A B, and Arithmetical complement of A C B we have the Log. of the side C B; now without transcribing the 1st. and 2nd. Logs. we have only to place the Log. sine of angle C B A under the product; the side A C will be got by adding every other line together; this mode may appear confused, but after a little practice there will be no fear of adding up the wrong lines.

The Arithmetical Complement of a sine will be found in the column of Co-secants by omitting the first unit.

The same holds good with the Log. Tangent, its complement will be found in the Column of Co-tangents.

INSTRUCTIONS FOR REDUCING LINES WHEN THE ANGLE OF ELEVATION OR DEPRESSION IS USED FOR THAT PURPOSE.

There are some Theodolites that have a scale on the back of the vertical arch, shewing the difference between the Hypothenuse and Base.

Should the Surveyor require such, and his Instrument unprovided with it, the following Table will answer the same purpose; in short, I am inclined to say better, as the Tables generally used for reducing Lines, are so erroneous, that with a long distance and a moderate angle, the quantity to be taken from the Hypothenuse to reduce it to the Horizon is far from the truth.

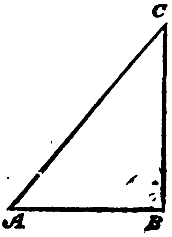
The difference would be but trifling in short distances, but in a mountainous Country, where the slopes are of a considerable length the difference will be augmented, as may be seen by the following Examples.

The scales and Tables generally used, give the number to be taken from each chain, according to the angle of Elevation or Depression, whereas the following Table gives the length of the Base when the Hypothenuse is one, also the difference between Hypothenuse and Base in 100; N. cosines may be used as multipliers for the length of the Hypothenuse, which product will give the Base or horizontal line, and the N. Sines used as multipliers for the length of the Hypothenuse, will produce the perpendicular B C.

The Table is simply the natural Sines and Cosines from one to ninety Degrees, which for most purposes will be found sufficient,

Example 1st. Let the Hypothenuse A C be 9000, and the angle of Elevation or Depression 5°, required the Horizontal line A B; here I find the number answering to 5° gives ·9961947 × 9000 = 8965·7, the length A B; by several Tables I find that 5° gives ·22 to one hundred, which for the above distance would produce 39·6, which in this case would make the Horizontal line 8960·4, in this distance here is a difference of 5·3.

Fig. 33.



In Mr. Burr's Treatise on Practical Surveying, six degrees gives ·6, now take this with a distance of (say) 12000, we shall have 72 of a difference between the hypothenuse and base; by the following Table, we shall have by multiplying the line by the number answering to 6° a base equal to 11934·2, and by the difference in column 3 we shall have the difference between the hypothenuse and base 65; now, 12000 — 65 = 11935, but taking Mr. Burr's number we shall have 12000 — 72 = 11928, between the two Tables a difference of 7.

There are some Tables so constructed that the difference between the hypothenuse and base is a certain number to 100, at a given angle, as if the Surveyor could make Hills to fit the angles.

I would not have introduced the following Table if it were not that some would prefer reducing the Lines measured down or up a steep hill by the Table, rather than level it in short portions as was before treated of.

It may be made serviceable for other purposes. For instance, if we wished to know the length of a degree of Longitude in any parallel of Latitude, we have only to multiply the natural cosine answering to the given Latitude by 60, the product will be the length of a degree in that Latitude.

Example. Required the Length of a degree of Longitude in Latitude 54·30, we find by the Table that the natural Cosine of 54·30 is ·58068 × 60 = 34·84 miles, nearly, the natural Cosine is calculated as a decimal.

As we have the N. Cosines of angles from 1 to 90, so at the same time we have the N. Sines of the Comp. of those angles; for instance the N. Cosine of 35° is the N. Sine of 55°, consequently some use may be made of the N. Sines as well as Cosines.

For example, if the side A C Fig. 33 was given with the angle C A B to find the height B C, we have only to multiply the line A C by the N. Sine of the angle C A B, and the length of B C will be obtained. Let the line A C be 1000, with an angle of 48° here the N. Sine of 48° is ·7431448, which being multiplied by 1000 we have only to remove the decimal point 3 places to the right, and we shall have the length of the line B C, 743·1448.

DEG.	N. COSINE.		DIFF.	DEG.	N. COSINE.		DIFF.
1	·9998477	89	.01	46	·6946584	44	30·53
2	·9993908	88	.06	47	·6819984	43	31·80
3	·9986295	87	·13	48	·6691306	42	33·08
4	·9975641	86	·24	49	·6560590	41	34·39
5	·9961947	85	.38	50	·6427876	40	35·72
6	·9945219	84	·54	51	·6293204	39	37·06
7	·9925462	83	·74	52	·6156615	38	38·43
8	·9902681	82	·97	53	·6018150	37	39·81
9	·9876883	81	1·23	54	·5877853	36	41·22
10	·9848078	80	1·52	55	·5735764	35	42·64
11	·9816272	79	1·83	56	·5591929	34	44·08
12	·9781476	78	2·18	57	·5446390	33	45·53
13	·9743701	77	2·56	58	·5299193	32	47·00
14	·9702957	76	2·97	59	·5150381	31	48·49
15	·9659258	75	3·40	60	·5000000	30	50·0
16	·9612617	74	3·87	61	·4848096	29	51·51
17	·9563048	73	4·36	62	·4694716	28	53·05
18	·9510565	72	4·89	63	·4539908	27	54·60
19	·9455186	71	5·44	64	·4383711	26	56·16
20	·9396926	70	6·03	65	·4226183	25	57·73
21	·9335804	69	6·64	66	·4067366	24	59·32
22	·9271839	68	7.28	67	·3907311	23	60·92
23	·9205049	67	7.94	68	·3746066	22	62·53
24	·9135455	66	8.64	69	·3583679	21	64·16
25	·9063078	65	9·37	70	·3420201	20	65·79
26	·8987940	64	10·12	71	·3255682	19	67·44
27	·8910065	63	10·90	72	·3090170	18	69.09
28	·8829476	62	11·70	73	·2923717	17	70·76
29	·8746197	61	12·54	74	·2756374	16	72·43
30	·8660254	60	13·40	75	·2588190	15	74·11
31	·8571673	59	14·28	76	·2419219	14	75·80
32	·8480481	58	15·20	77	·2249511	13	77·50
33	·8386706	57	16.13	78	·2079117	12	79·20
34	·8290376	56	17·09	79	·1908090	11	80·91
35	·8191520	55	18·08	80	·1736482	10	82·63
36	·8090170	54	19·09	81	·1564345	9	84·35
37	·7986355	53	20·13	82	·1391731	8	86·08
38	·7880108	52	21·19	83	·1218693	7	87·81
39	·7771460	51	22·28	84	·1045285	6	89·54
40	·7660444	50	23·39	85	·0871557	5	91·28
41	.7547096	49	24·52	86	·0697565	4	93·02
42	·7431448	48	25·68	87	·0523360	3	94·76
43	·7313537	47	26·86	88	·0348995	2	96·51
44	·7193398	46	28·06	89	·0174524	1	98·25
45	·7071068	45	29·28	90		0	0
	N. SINE.	DEG.			N. SINE.	DEG.	

The foregoing table does not contain minutes, therefore if we have to use degrees and minutes, we must take the difference between the two degrees and divide it according to the number of minutes; should we require the N. Cosine of 54° 30′, it is done thus: 54° gives ·5877853, and 55° gives ·5735764, ½ of their difference taken from the greatest or added to the least, will give the No. very near the truth.

Where other Tables are not at hand it may be found of service, to the Traveller who wishes to ascertain hastily the height of a Hill, &c., it may be found, (if he has the means of measuring an angle of Elevation or Depression,) by pacing the Slope and finding the length as before directed, and will come as near the truth as a Survey made by the pocket Sextant and pacing the lines.

Before I conclude, another example will be given to illustrate the use of the Table.

Suppose we have the acclivity or declivity of a hill, road, or any other inclined surface, and it is desirable to know how much it falls or rises in any given distance, by help of the Table it is done as follows: multiply the given length by the natural sine of the given angle, and we shall have the fall or rise required. For instance, if the line A C fig. 33, be a slope of 12°, and the fall at 30 feet is required, the calculation will be thus, N. Sine 12° $=$ ·2079117 × 30 (the given length) $=$ 6·297, consequently the fall in 30 feet will be 6ft. 3½ in. nearly

Persons whose duty it may be to find heights and distances in haste, would do well to make a Table of N. Sines, Cosines, and Tangents, to suit the description of Instruments they may have to use; if a Pocket Sextant, degrees will be as near as that Instrument is capable of measuring; such a Table might be pasted in the crown of a Cap or Sketching Case, by having such, the distance to inaccessable objects may be found with little trouble where a few yards are of no consequence.

FINIS.

J. Hickmutt, Printer, Troy Town, Rochester.